Generalized Autoregressive Moving Average Models: An Application to GDP in Malaysia

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Abstract

Gross Domestic Product (GDP) per capita is often used as an indicator of standard of living in an economy. GDP per capita observed over the years can be modelled using time series models. A new class of Generalized Autoregressive Moving Average (GARMA) namely GARMA (1, 2; δ, 1) has been introduced in the time series literature to reveal some hidden features in time series. In this paper, GARMA (1, 2; δ, 1) model and ARMA (1, 1) model are fitted in the GDP growth data of Malaysia which has been observed from 1955 to 2009. The parameter estimation methods considered include the Hannan Rissanen Algorithm (HRA), Whittle Estimation (WE) and Maximum Likelihood Estimation (MLE). Point forecasts also have been done and the performance of GARMA (1, 2; δ, 1) and ARMA (1, 1) and the estimation methods are discussed.

Keywords: Hannan-Rissanen Algorithm Estimates, Maximum Likelihood Estimation, Whittle's Estimates,

1 Introduction

Time series is a set of well-defined data items collected at successive points at uniform time intervals (Prapanna et al. (2014)). The goal of time series analysis is to predict a series that contains a random component. If this random component is stationary, then we can develop powerful techniques to forecast its future values (Brockwell and Davis (2002)). Forecasting is important in the fields like finance, meteorology, industry and so forth (Chen et al. (2014)).

It is known that the modelling of time series with changing frequency components is important in many applications, especially in financial data. These type of time series cannot be identified using the existing standard time series techniques. However, one may propose the same classical model for all these cases. This may produce poor forecast values (Peiris et al. (2004)). Due to that, Peiris introduced a new class of Autoregressive Moving Average (ARMA) type models with indices called Generalized ARMA (GARMA) to describe data with different frequency components (Peiris (2003)).

Firstly, Peiris has introduced Generalised Autoregressive (GAR(1)) model, and followed by the Generalised Moving Average (GMA(1)) model (Peiris (2003), Peiris et al. (2004)). More recently, the GARMA (1, 1; 1, δ) model has been considered (Pillai et al. (2009)). In addition, Shitan and Peiris studied the behaviour of the process
GARMA(1, 1; δ, 1) (Shitan and Peiris (2011)).

The GARMA (1, 1; 1, δ) and GARMA(1, 1; δ, 1) models can be further generalised as GARMA (1, 1; δ_1, δ_2) and some properties of this model have been established (Pillai et al. (2012)). All these models have been shown to be useful in modelling time series data. It is interesting to note that the GARMA model can be further expanded to GARMA (1, 2; δ, 1).

These GARMA models give a better forecast compared to traditional ARMA models. This will be supported by the modelling of the Gross Domestic Product per capita of Malaysia.

The Gross Domestic Product (GDP), the Gross National Product (GNP) and the Net National Income (NNI), all are indicators of a country's economic power. Nevertheless, in almost all the countries, the GDP per capita is used as a benchmark for measuring the nation's economic progress. GDP is the measure of the market value of all goods and services produced within a country during a specified period. GDP per capita is the share of individual members of the population to the annual GDP. It is calculated by dividing real or nominal GDP by the number of population per year. GDP per capita is an indicator of the average standard of living of individual members of the population. An increase in the GDP per capita signifies national economic growth. The GDP per capita observed over years can be modelled using time series models.

The objective of this paper is to compare the performance of the ARMA (1, 1) model and GARMA (1, 2; δ, 1) model besides comparing the three estimation methods. In Section 2, we illustrate the applications of ARMA (1, 1) and GARMA (1, 2; δ, 1) modelling to GDP data set. Finally, the conclusions are drawn in Section 3.

2 Application of GARMA Modelling to GDP data set

In this section, ARMA (1, 1) and GARMA (1,2; δ, 1) modelling are given.

2.1 Stationary Data

The GDP of Malaysia was obtained from the official website of Department of Statistics Malaysia, National Accounts, consisting of yearly observations from 1955 to 2009. Figure 1 shows the time series plot of the GDP of Malaysia from 1955 to 2009 and it is quite apparent that it is a nonstationary time series. Many observed time series, however, are not stationary. In particular, most economic and business series exhibit time-changing levels and/or variances (Abraham and Ledolter (1983)).

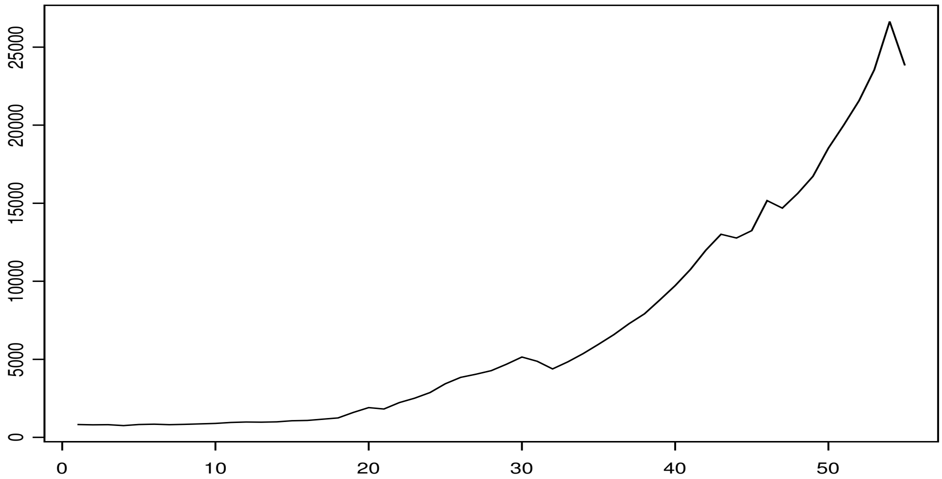

Figure 1: GDP per capita of Malaysia from 1955 to 2009

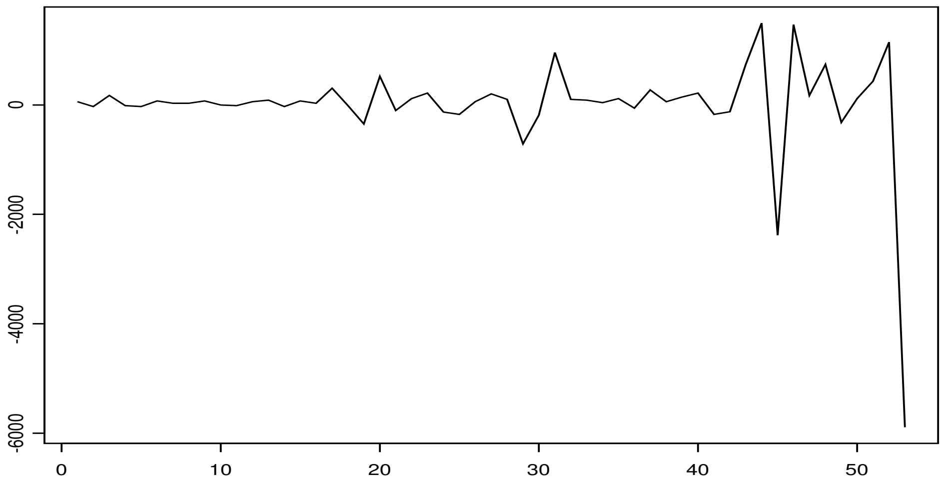

Figure 2: GDP per capita of Malaysia which was twice differenced at lag 1 and mean corrected

In order to achieve stationarity, the data set was twice-differenced at lag 1 and mean corrected and a plot of this is depicted in Figure 2 and it can be written as $Y_t = (1 - B)(1 - B)(X_t - 52.7925)$.

Then the computer programs were written to model the GDP per capita of Malaysia using ARMA (1, 1) and GARMA(1, 2; δ, 1) model.

2.2 ARMA(1, 1)

The objective of this section is to illustrate the modelling of the GDP data of Malaysia using ARMA(1, 1). The preliminary estimation of the parameters of this model has been done using the Hannan-Rissanen Algorithm Estimator. We fit a standard ARMA(1, 1) model for the differenced data and the following results were obtained.

The Hannan-Rissanen Algorithm estimation is obtained for the ARMA(1, 1) model and the fitted model is $(1 - 0.2091B)Y_t = (1 - 0.4938B)Z_t$, where $Z_t \sim WN(0, 884910)$. On the other hand, the ARMA(1, 1) fitted models are, $(1 - 0.2923B)Y_t = (1 - 0.3488B)Z_t$, where $Z_t \sim WN(0, 66132)$, by the Whittle's estimation method and $(1-0.9830B)Y_t = (1+0.0103B)Z_t$, where $Z_t \sim WN(0, 884910)$, by the Maximum Likelihood Estimation method.

Using the above fitted models, point forecasts for the GDP data set for the next six time periods are shown in Table 1. The point forecasts obtained from MLE method gives the best answer.

Table 1: Actual and forecast values for GDP data using ARMA (1, 1) Model

Step	Actual value	Forecast value using HRA	Forecast value using WE	Forecast value using MLE
1	18531	4446	1214	16448
2	19996	4940	1334	18238
3	21563	5320	1446	19675
4	23544	5738	1560	21217
5	26639	6270	1700	23169
6	23826	7111	1907	26222

2.3 GARMA(1, 2; δ, 1)

GARMA(1, 2; δ, 1) model was fitted to the GDP data set that has been differenced and mean corrected. The Hannan-Rissanen Algorithm estimation is obtained for the GARMA(1, 2; δ, 1) model and the fitted model is $(1 - 0.9237B)^{0.9237}Y_t = (1 - 0.1443B + 0.9521B^2)Z_t$ where $Z_t \sim WN(0, 1827225)$. On the other hand, the GARMA(1, 2; δ, 1) fitted model is, $(1 - 0.9920B)^{0.2468}Y_t = (1 - 0.3845B - 0.0604B^2)Z_t$ where $Z_t \sim WN(0, 12943)$, by the Whittle's estimation The GARMA(1, 2; δ, 1) fitted model is, $(1-0.8665B)^{0.9982}Y_t = (1+0.0012B+0.0004B^2)Z_t$, where $Z_t \sim WN(0, 1827225)$, by the Maximum Likelihood Estimation method.

Using the above fitted models, point forecasts for the GDP data set for the next six time periods are shown in Table 2. It can be seen from Table 2 that all the point forecasted values through HRA and MLE estimation give a closer reading to the actual values compared to WE estimation. GARMA(1, 2; δ, 1) results are closer to the true values than the traditional ARMA (1, 1) model.

Table 2: Actual and forecast values for GDP data using GARMA(1, 2; δ, 1)

Step	Actual value	Forecast value using HRA	Forecast value using WE	Forecast value using MLE
1	18531	16538	14342	16553
2	19996	18238	15368	18264
3	21563	19738	16502	19762
4	23544	21295	17741	21320
5	26639	23214	19162	23244
6	23826	26136	20993	26181

3 Conclusion

The objective of our study was to compare the performance of ARMA (1, 1) and GARMA(1, 2; δ, 1). In addition, we have evaluated the performance of the three estimators based on HRA, WE and MLE. It appears from this study, that the MLE estimation procedure is relatively good for ARMA (1, 1). HRA and MLE estimation give a closer reading to the actual values compared to WE estimation for GARMA (1, 2; δ, 1) model. GARMA (1, 2; δ, 1) performs better than ARMA (1, 1) for all the estimation methods. We have successfully illustrated the superiority, usefulness and applicability of the GARMA (1, 2; δ, 1) model using the GDP data set.

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